MAT 102	VECTOR CALCULUS, DIFFERENTIAL EQUATIONS AND TRANSFORMS	CATEGORY	L	T	P	CREDIT	Year of Introduction
		BSC	3	1	0	4	2019

Preamble: This course introduces the concepts and applications of differentiation and integration of vector valued functions, differential equations, Laplace and Fourier Transforms. The objective of this course is to familiarize the prospective engineers with some advanced concepts and methods in Mathematics which include the Calculus of vector valued functions, ordinary differential equations and basic transforms such as Laplace and Fourier Transforms which are invaluable for any engineer's mathematical tool box. The topics treated in this course have applications in all branches of engineering.

Prerequisite: Calculus of single and multi variable functions.

Course Outcomes: After the completion of the course the student will be able to

CO 1	Apply the concept of vector functions and learn to work with conservative vector field
CO 2	Apply computing integrals of scalar and vector field over surfaces in three-dimensional space.
CO 3	Solve homogeneous and non-homogeneous linear differential equation with constant coefficients
CO 4	Apply Laplace transforms to solve physical problems arising in engineering
CO 5	Apply Fourier transforms to solve physical problems arising in engineering

Mapping of course outcomes with program outcomes

	PO 1	PO 2	PO 3	PO 4	PO 5	PO 6	PO 7	PO 8	PO 9	PO 10	PO 11	PO 12
CO 1	3											
CO 2	3											
CO 3	3											
CO 4	3											
CO 5	3											
CO 6	3											

Assessment Pattern

Bloom's Category	Continuous Assessment Tests		End Semester Examination (Marks)
	Test 1 (Marks	Test 2 (Marks)	
Remember	10	10	20
Understand	20	20	40
Apply	20	20	40
Analyse			
Evaluate			

Create			

Mark distribution

Total Marks	**CIE** (Marks)	**ESE** (Marks)	**ESE Duration**
150	50	100	3 hours

Continuous Internal Evaluation Pattern:

Attendance : 10 marks
Continuous Assessment Test (2 numbers) : 25 marks
Assignment/Quiz/Course project : 15 marks
Assignments: Assignment should include specific problems highlighting the applications of the methods introduced in this course in science and engineering.

End Semester Examination Pattern: There will be two parts; Part A and Part B. Part A contain 10 questions with 2 questions from each module, having 3 marks for each question. Students should answer all questions. Part B contains 2 questions from each module of which student should answer any one. Each question can have maximum 2 sub-divisions and carry 14 marks.

Course Level Assessment Questions

Course Outcome 1 (CO1): Apply the concept of vector functions and learn to work with conservative vector field

1. How would you calculate the speed, velocity and acceleration at any instant of a particle moving in space whose position vector at time $t$is $\boldsymbol{r}(t)$?

2. Find the work done by the force field $F = (e^x - y^3)\boldsymbol{i} + (\cos y + x^3)$on a particle that travels once around the unit circle centred at origin having radius 1.

3. When do you say that a vector field is conservative? What are the implications if a vector field is conservative?

Course Outcome 2 (CO2): Apply computing integrals of scalar and vector field over surfaces in three-dimensional space.

1. Write any one application each of line integral, double integral and surface integral.

2. Use the divergence theorem to find the outward flux of the vector field $F(x, y\,, z) = z\boldsymbol{k}$across the

$$x^2 + y^2 + z^2 = a^2$$

3. State Greens theorem. Use Green's theorem to express the area of a plane region bounded by a curve as a line integral.

Course Outcome 3 (CO3): Solve homogeneous and non-homogeneous linear differential equation with constant coefficients

1. If $y_1(x)$ and $y_2(x)$ are solutions of $y'' + py' + qy = 0$, where p, q are constants, show that

   $y_1(x) + y_2(x)$ is also a solution.

2. Solve the differential equation $y'' + y = 0.001x^2$ using method of undetermined coefficient.

3. Solve the differential equation of $y''' - 3y'' + 3y' - y = e^x - x - 1$.

Course Outcome 4 (CO4): Apply Laplace transforms to solve physical problems arising in engineering

1. What is the inverse Laplace Transformof $(s) = \frac{3s-13}{s^2+2s+401}$?

2. Find Laplace Transform of Unit step function.

3. Solve the differential equation of $y'' + 9y = \delta\left(t - \frac{\pi}{2}\right)$? Given $y(0) = 2,\ y'(0) = 0$

Course Outcome 5(CO5): Apply Fourier transforms to solve physical problems arising in engineering

1. Find the Fourier integral representation of function defined by $f(x) = e^{-x}$ for $x > 0$ and $f(x) = 0$ for $x < 0$.

2. What are the conditions for the existence of Fourier Transform of a function $f(x)$?

3. Find the Fourier transform of $f(x) = 1$ for $|x| < 1$ and $f(x) = 0$ otherwise.

Model Question paper

QP CODE: PAGES:3

Reg No:_______________

Name :_______________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY SECOND SEMESTER B.TECH DEGREE EXAMINATION,

MONTH & YEAR

VECTOR CALCULUS, DIFFERENTIAL EQUATIONS AND TRANSFORMS

Course Code: MAT 102

Max. Marks: 100 **Duration: 3 Hours**

(2019-Scheme)

(Common to all branches)

PART A

(Answer all questions. Each question carries 3 marks)

(Answer all questions. Each question carries 3 marks)

1. Is the vector $\boldsymbol{r}$ **where** $\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$ conservative. Justify your answer.
2. State Greens theorem including all the required hypotheses
3. What is the outward flux of $\boldsymbol{F}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$ across any unit cube.
4. What is the relationship between Green's theorem and Stokes theorem?
5. Solve $y'' + 4y' + 2.5y = 0$
6. Does the function $y = C_1 \cos x + C_2\, sinx$form a solution of$y'' + y = 0$?. Is it the general solution? Justify your answer.
7. Find the Laplace transform of $e^{-t} \sinh 4t$
8. Find the Laplace inverse transform of $\frac{1}{s\,(s^2+\omega^2)}$.
9. Given the Fourier transform $\frac{1}{\sqrt{2}} e^{-\frac{\omega^2}{4}}$ of $f(x) = e^{-x^2}$, find the Fourier transform of xe^{-x^2}
10. State the convolution theorem for Fourier transform

PART B

(Answer one full question from each module. Each full question carries 14 marks)

MODULE 1

11a) Prove that the force field $\boldsymbol{F} = e^y\boldsymbol{i} + xe^y\boldsymbol{j}$is conservative in the entire xy-plane

b**)** Use Greens theorem to find the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

12 a) Find the divergence of the vector field$\boldsymbol{F} = \frac{c}{(x^2+y^2+z^2)^{3/2}}(x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k})$

b) Find the work done by the force field$\boldsymbol{F}(x, y, z) = xy\boldsymbol{i} + yz\boldsymbol{j} + xz\boldsymbol{k}$along C where

C is the curve$\boldsymbol{r}(t) = t\boldsymbol{i} + t^2\boldsymbol{j} + t^3\boldsymbol{k}$

MODULE II

13 a) Use divergence theorem to find the outward flux of the vector field

$\boldsymbol{F} = 2x\boldsymbol{i} + 3y\boldsymbol{j} + z^3\boldsymbol{k}$ across the unit cube bounded by or $x = 0, y = 0, z = 0, x = 1, y = 1, z = 1$

b) Find the circulation of $\boldsymbol{F} = (x - z)\boldsymbol{i} + (y - x)\boldsymbol{j} + (\boldsymbol{z} - \boldsymbol{xy})\boldsymbol{k}$ using Stokes theorem around the triangle with vertices $A(1,0,0)$, $B(0,2,0)$and $C(0,0,1)$

14 a) Use divergence theorem to find the volume of the cylindrical solid bounded by $x^2+4x+y^2=7, z=-1, z=4$, given the vector field $\boldsymbol{F}=\boldsymbol{x}i+\boldsymbol{y}j+\boldsymbol{z}k$ across surfaceof the cylinder

b) Use Stokes theorem to evaluate $\int_C \boldsymbol{F}.\boldsymbol{dr}$ where $\boldsymbol{F}=x^2\boldsymbol{i}+3x\boldsymbol{j}-y^3\boldsymbol{k}$ where Cis the circle $x^2+y^2=1$ in the xy- plane with counterclockwise orientation looking down the positive z-axis

MODULE III

15 a) Solve $y''+4y'+4y=x^2+e^{-x}\cos x$

b) Solve $y'''-3y''+3y'-y=e^x-x-1$

16 a)solve $\boldsymbol{y}'''+\boldsymbol{3}y''+3y'+y=30e^{-x}$ given $y(0)=3, y'(0)=-3\,,\ \ y''(0)=-47$

b)Using method of variation of parameters, solve $y''+y=sec\,x$

MODULE IV

17 a) Find the inverse Laplace transform of $F(s)=\frac{2(e^{-s}-e^{-3s})}{s^2-4}$

b)Solve the differential equation $y''+16y=4\delta(t-3\pi); y(0)=2, y'(0)=0$ using Laplace transform

18 a) Solve $y''+3y'+2y=f(t)$ where $f(t)=1$ for $0<t<1$ and $f(t)=1$ for $t>1$ using Laplace transform

b) Apply convolution theorem to find the Laplace inverse transform of $\frac{1}{s^2(s^2+\omega^2)}$

MODULE V

19 a) Find the Fourier cosine integral representation for $f(x)=e^{-kx}$ for $x>0$ and $k>0$ and hence evaluate $\int_0^\infty \frac{\cos wx}{k^2+w^2}$ the function

b) Does the Fourier sine transform $f(x)=x^{-1}\sin x$ for $0<x<\infty$ exist? Justify your answer

20 a) Find the Fourier transform of $f(x)=|x|$ for $|x|<1$ and $f(x)=0$ otherwise

b) Find the Fourier cosine transform of $f(x)=e^{-ax}$ for a> 0

Syllabus

Module 1 (Calculus of vector functions)

(Text 1: Relevant topics from sections 12.1, 12.2, 12.6, 13.6, 15.1, 15.2, 15.3)

Vector valued function of single variable, derivative of vector function and geometrical interpretation, motion along a curve-velocity, speed and acceleration. Concept of scalar and vector fields , Gradient and its properties, directional derivative , divergence and curl, Line integrals of vector fields, work as line integral, Conservative vector fields , independence of path and potential function(results without proof).

Module 2 (Vector integral theorems)

(Text 1: Relevant topics from sections 15.4, 15.5, 15.6, 15.7, 15.8)

Green's theorem (for simply connected domains, without proof) and applications to evaluating line integrals and finding areas. Surface integrals over surfaces of the form z = g(x, y), y = g(x, z) or x = g(y, z) , Flux integrals over surfaces of the form z = g(x, y), y = g(x, z) or x = g(y, z), divergence theorem (without proof) and its applications to finding flux integrals, Stokes' theorem (without proof) and its applications to finding line integrals of vector fields and work done.

Module- 3 (Ordinary differential equations)

(Text 2: Relevant topics from sections 2.1, 2.2, 2.5, 2.6, 2.7, 2.10, 3.1, 3.2, 3.3)

Homogenous linear differential equation of second order, superposition principle,general solution, homogenous linear ODEs with constant coefficients-general solution. Solution of Euler-Cauchy equations (second order only).Existence and uniqueness (without proof). Non homogenous linear ODEs-general solution, solution by the method of undetermined coefficients (for the right hand side of the form x^n,e^{kx},$sinax$,$cosax$,$e^{kx}sinax e^{kx}cosax$and their linear combinations), methods of variation of parameters. Solution of higher order equations-homogeneous and non-homogeneous with constant coefficient using method of undetermined coefficient.

Module- 4 (Laplace transforms)

(Text 2: Relevant topics from sections 6.1,6.2,6.3,6.4,6.5)

Laplace Transform and its inverse ,Existence theorem (without proof) , linearity,Laplace transform of basic functions, first shifting theorem, Laplace transform of derivatives and integrals, solution of differential equations using Laplace transform, Unit step function, Second shifting theorems. Dirac delta function and its Laplace transform, Solution of ordinary differential equation involving unit step function and Dirac delta functions .Convolution theorem(without proof)and its application to finding inverse Laplace transform of products of functions.

Module-5 (Fourier Tranforms)

(Text 2: Relevant topics from sections 11.7,11.8, 11.9)

Fourier integral representation, Fourier sine and cosine integrals. Fourier sine and cosine transforms, inverse sine and cosine transform. Fourier transform and inverse Fourier transform, basic properties. The Fourier transform of derivatives. Convolution theorem (without proof)

Text Books

1. H. Anton, I. Biven S.Davis, "Calculus", Wiley, 10th edition, 2015.
2. Erwin Kreyszig, "Advanced Engineering Mathematics", Wiley, 10th edition, 2015.

Reference Books

1. J. Stewart, Essential Calculus, Cengage, 2nd edition, 2017
2. G.B. Thomas and R.L. Finney, Calculus and Analytic geometry, 9 th Edition, Pearson,Reprint, 2002.
3. Peter O Neil, Advanced Engineering Mathematics, 7th Edition, Thomson, 2007.
4. Louis C Barret, C Ray Wylie, "Advanced Engineering Mathematics", Tata McGraw Hill, 6th edition, 2003.
5. VeerarajanT."Engineering Mathematics for first year", Tata McGraw - Hill, 2008.
6. B.S. Grewal, Higher Engineering Mathematics, Khanna Publishers, 36th edition , 2010.
7. Srimanta Pal, Subodh C. Bhunia, "Engineering Mathematics", Oxford University Press, 2015.
8. Ronald N. Bracewell, "The Fourier Transform and its Applications", McGraw – Hill International Editions, 2000.

Course Contents and Lecture Schedule

No	Topic	No. of Lectures
1	**Calculus of vector functions (9 hours)**	
1.1	Vector valued function of a scalar variable - derivative of vector valued function of scalar variable t-geometrical meaning	2
1.2	Motion along a curve-speed , velocity, acceleration	1
1.3	Gradient and its properties, directional derivative , divergent and curl	3
1.4	Line integrals with respect to arc length, line integrals of vector fields. Work done as line integral	2

1.5	Conservative vector field, independence of path, potential function	1
2	**Vector integral theorems**(**9 hours**)	
2.1	Green's theorem and it's applications	2
2.2	Surface integrals , flux integral and their evaluation	3
2.3	Divergence theorem and applications	2
2.4	Stokes theorem and applications	2
3	**Ordinary Differential Equations** (**9 hours**)	
3.1	Homogenous linear equation of second order, Superposition principle, general solution	1
3.2	Homogenous linear ODEs of second order with constant coefficients	2
3.3	Second order Euler-Cauchy equation	1
3.4	Non homogenous linear differential equations of second order with constant coefficient-solution by undetermined coefficients, variation of parameters.	3
3.5	Higher order equations with constant coefficients	2
4	**Laplace Transform** (**10 hours**)	
4.1	Laplace Transform , inverse Transform, Linearity, First shifting theorem, transform of basic functions	2
4.2	Transform of derivatives and integrals	1
4.3	Solution of Differential equations, Initial value problems by Laplace transform method.	2
4.4	Unit step function --- Second shifting theorem	2
4.5	Dirac Delta function and solution of ODE involving Dirac delta function	2
4.6	Convolution and related problems.	1
5	**Fourier Transform** (**8 hours**)	
5.1	Fourier integral representation	1
5.2	Fourier Cosine and Sine integrals and transforms	2
5.3	Complex Fourier integral representation, Fourier transform and its inverse transforms, basic properties	3

5.4	Fourier transform of derivatives, Convolution theorem	2